ADVANCED MATHEMATICS



Final Exam - December 2013

Name:______ Group: _____ NIU: ______ Group: _____ Grade:

Instructions: This exam consists of six questions. You have two hours to give a reasoned answer to all the exercises. Write the quiz entirely in ink. Calculators are not permitted.

1 Determine for which values of the parameter $a \in \mathbb{R}$ the matrix A is diagonalizable. (20 points)

$A = \left(\begin{array}{rrr} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & a \end{array}\right)$

- 2 Suppose that in a given market with a single commodity the demand function is D(P) = 4 P and the supply function is S(P) = -2 + 2P, where P > 0 denotes the unitary price of the good. Assume that time is discrete and that the market follows the dynamics of the Cobweb Model, that is, $S(P_t) = D(P_{t+1})$ for every t. (15 points)
 - (a) Obtain the expression of P_t when $P_0 = 4$.
 - (b) Calculate the equilibrium \overline{P} .
 - (c) Analyze the behavior of the price in the long run.

3 Consider the following system of equations (20 points)

$$X_{t+1} = \left(\begin{array}{cc} 2 & -2\\ 0 & 1 \end{array}\right) X_t.$$

- (a) Obtain the solutions of the previous system.
- (b) Calculate the equilibrium \overline{X} .
- (c) Is the equilibrium \overline{X} globally asymptotically stable?

4 Solve the following differential equation: $(t^2 - 1)x' = -2tx$. (15 points)

5 Solve the following equation: $x'' - 5x' + 6x = e^{2t}$. (20 points)

- 6 Consider the differential equation x' = f(x). The following picture shows the trajectories of the solutions of such an equation. (10 points)
 - (a) Identify the equilibria.
 - (b) Study the stability of those equilibria.
 - (c) Draw a sketch of the phase diagram that would correspond to this situation.

