## ADVANCED MATHEMATICS

Final Exam - December 2013
$\qquad$ Group: $\qquad$
Grade:

Instructions: This exam consists of six questions. You have two hours to give a reasoned answer to all the exercises. Write the quiz entirely in ink. Calculators are not permitted.

1 Determine for which values of the parameter $a \in \mathbb{R}$ the matrix $A$ is diagonalizable. ( 20 points)

$$
A=\left(\begin{array}{lll}
2 & 0 & 0 \\
0 & 1 & 3 \\
0 & 0 & a
\end{array}\right)
$$

2 Suppose that in a given market with a single commodity the demand function is $D(P)=4-P$ and the supply function is $S(P)=-2+2 P$, where $P>0$ denotes the unitary price of the good. Assume that time is discrete and that the market follows the dynamics of the Cobweb Model, that is, $S\left(P_{t}\right)=D\left(P_{t+1}\right)$ for every $t$. (15 points)
(a) Obtain the expression of $P_{t}$ when $P_{0}=4$.
(b) Calculate the equilibrium $\bar{P}$.
(c) Analyze the behavior of the price in the long run.

3 Consider the following system of equations (20 points)

$$
X_{t+1}=\left(\begin{array}{cc}
2 & -2 \\
0 & 1
\end{array}\right) X_{t}
$$

(a) Obtain the solutions of the previous system.
(b) Calculate the equilibrium $\bar{X}$.
(c) Is the equilibrium $\bar{X}$ globally asymptotically stable?

44 Solve the following differential equation: $\left(t^{2}-1\right) x^{\prime}=-2 t x$. ( 15 points)

55 Solve the following equation: $x^{\prime \prime}-5 x^{\prime}+6 x=e^{2 t}$. (20 points)

56 Consider the differential equation $x^{\prime}=f(x)$. The following picture shows the trajectories of the solutions of such an equation. (10 points)
(a) Identify the equilibria.
(b) Study the stability of those equilibria.
(c) Draw a sketch of the phase diagram that would correspond to this situation.


